# LIMITLESS MIND 

LEARN, LEAD, AND LIVE WITHOUT BARRIERS

JO BOALER

[^0]


## The Steps of Struggle

Which step have you reached today?






LANG CHEN

| First fold <br> the paper <br> in half | Then <br> in half <br> again | Then <br> fold a <br> triangle | Then <br> open the <br> paper |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |

## From this:

Division 6-12
Name:

| $9 \longdiv { 8 1 }$ | $1 1 \longdiv { 1 2 1 }$ | $7 \longdiv { 2 1 }$ | $1 0 \longdiv { 1 0 }$ | $1 0 \longdiv { 1 0 }$ |
| :---: | :---: | :---: | :---: | :---: |
| $1 0 \longdiv { 5 0 }$ | $7 \longdiv { 4 9 }$ | $1 0 \longdiv { 5 0 }$ | $9 \longdiv { 2 7 }$ | $8 \longdiv { 6 4 }$ |
| $1 0 \longdiv { 9 0 }$ | $9 \longdiv { 6 3 }$ | $8 \longdiv { 9 6 }$ | $7 \longdiv { 7 7 }$ | $1 0 \longdiv { 9 0 }$ |
| $1 2 \longdiv { 3 6 }$ | $1 1 \longdiv { 1 1 }$ | $1 1 \longdiv { 1 1 }$ | $1 2 \longdiv { 1 3 2 }$ | $6 \longdiv { 3 0 }$ |
| $6 \longdiv { 5 4 }$ | $1 1 \longdiv { 5 5 }$ | $1 2 \longdiv { 8 4 }$ | $1 1 \longdiv { 5 5 }$ | $9 \longdiv { 4 5 }$ |

To this:


These are the seven dots:


These were their twenty-four ways of seeing the dots:

|  |  | $\cdots$ | - | > | + | + |  | 8 |  |  | . 8 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1-X |  | V | を | - | - | 兰 |  |  | - | x |  |

Multiplying by 12
NAME

| 2 | 12 | 6 | 7 | 6 | 12 | 4 | 8 | 2 | 5 | 12 | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| x12 | +12 | +12 | $\times 12$ | +12 | +12 | $\times 12$ | +12 | +12 | +12 | +12 | $\times 12$ |
| 9 | 4 | 12 | 2 | 3 | 3 | 6 | 4 | 11 | 6 | 7 | 2 |
| $\times 12$ | +12 | $\times 12$ | $\times 12$ | +12 | +12 | $\times 12$ | +12 | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ |
| 1 | 8 | 5 | 12 | 9 | 7 | 11 | 6 | 2 | 2 | 7 | 12 |
| $\times 12$ | $\times 12$ | $\times 12$ | x12 | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | x12 | $\times 12$ | $\times 12$ | $\times 12$ |
| 7 | 5 | 1 | 12 | 8 | 6 | 8 | 3 | 0 | 6 | 4 | 2 |
| $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ |
| 5 | 12 | 4 | 2 | 6 | 11 | 4 | 9 | 3 | 8 | 3 | 2 |
| +12 | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | +12 | +12 | $\times 12$ | $\times 12$ | $\times 12$ |
| 6 | 4 | 12 | 12 | 12 | 0 | 9 | 4 | 8 | 5 | 2 | 7 |
| $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ |
| 5 | 1 | 8 | 12 | 7 | 4 | 12 | 5 | 9 | 1 | 3 | 7 |
| $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | $\times 12$ | +12 | $\times 12$ | $\times 12$ | +12 | $\times 12$ |
| 8 | 9 | 5 | 5 | 6 | 11 | 7 | 3 | 6 | 5 | 8 | 5 |
| $\times 12$ | +12 | +12 | +12 | +12 | +12 | $\times 12$ | +12 | +12 | +12 | +12 | +12 |

Goal $\qquad$ Number Correct $\qquad$


Here are six different ways of calculating $18 \times 5$ (there are more) with their visual representations.

| $\begin{aligned} 20 \times 5 & =100 \\ 2 \times 5 & =10 \\ 100-10 & =90 \end{aligned}$ | $18 \times 5=9 \times 10$ |  |
| :---: | :---: | :---: |
|  | $\begin{aligned} 18 \times 2 & =36 \\ 18 \times 2 & =36 \\ 18 \times 1 & =18 \\ 36+36+18 & =90 \end{aligned}$ <br> 18 | $\begin{aligned} & 18 \times 10=180 \\ & 180 \div 2=90 \end{aligned}$ |

Raindrops are pushed by wind above the freezing



Visual Representations of Number Patterns


Students Plot the Stopping Times of Each Number

# RESOURCES to Help Change Mindsets and Approaches 

## Four Boosting Messages for Students: <br> https://www.youcubed.org/resources/four-boosting -messages-jo-students/

Free Online Class, in English and Spanish, to Improve Students' Mindset and Approach to Mathematics:
https://www.youcubed.org/online-student-course/

A Range of Mindset Videos for Students:
https://www.youcubed.org/resource/mindset-boosting -videos/

Rethinking Giftedness Film:
https://www.youcubed.org/rethinking-giftedness-film/

## Different Experiences with Maths Facts Film: <br> https://www.youcubed.org/resources/different <br> -experiences-with-math-facts/

Visual Creative Maths Tasks:
https://www.youcubed.org/tasks/

# Free Downloadable Posters: <br> https://www.youcubed.org/resource/posters/ 

# Two Online Courses for Mathematics Teachers and Parents: https://www.youcubed.org/online-teacher-courses/ 

K-8 Book Series:
https://www.youcubed.org/resource/k-8-curriculum/

A Range of Short, Readable News Articles on the Book's Ideas:
https://www.youcubed.org/resource/in-the-news/

## APPENDIX I

## Examples of Numerical and Visual Approaches to Math Problems

Here are two standard math questions with visual solutions. These are the types of questions that may have produced anxiety and math hatred in school, with good reason. I have written at length about the damage of fake word problems and contexts that students are meant to partly believe, while ignoring everything they know about the real situation. But please have a look at the different ways of solving them, as an illustration of what is possible when we think visually.

This problem is adapted from one used by a wonderful mathematics educator, Ruth Parker. She poses this question:

A man wants to buy $1 / 4$ of a pound of turkey. He goes into a shop that gives him 3 slices that weigh $1 / 3$ of a pound.
What proportion of the 3 slices does he need?

| A numerical approach: | A visual approach: |
| :---: | :---: |
| 3 slices $=1 / 3$ pound | $\bigcirc \bigcirc=1 / 3$ pound |
| $x$ slices $=1 / 4$ pound | $\bigcirc \bigcirc$ |
| $1 / 3 x=3 / 4$ | $\begin{aligned} & \bigcirc \bigcirc \bigcirc=1 / 3 \text { pound } \\ & \bigcirc \bigcirc \bigcirc \end{aligned}$ |
| $x=9 / 4$ |  |

The second is one of those awful, unrealistic word problems that fill mathematics textbooks:

Jo and Tesha each have a number of cards in the ratio 2:3.
Tesha and Holly have a number of cards in the ratio 2:1. If
Tesha has 4 more cards than Jo, how many cards does Holly
have? Give an answer and briefly explain your reasoning.

## A numerical approach:

Jo and Tesha 2:3
Tesha and Holly 2:1
Jo and Tesha's cards are divided into 5 , with a 2:3 ratio.

Tesha has $1 / 5$ more than Jo. Tesha has 4 more cards.
$1 / 5=4$
$1=20$
So together they have 20 cards.

Jo has $2 / 5 \times 20$ and
Tesha has $3 / 5 \times 20$.
Jo has 8 and Tesha has 12.
Tesha and Holly are 2:1, so Holly has 6 cards.

## A visual approach:

Jo \& Tesha 2:3 Tesha \& Holly 2:1

(these are ratios, so we don't know the value-yet)

Tesha
Holly

:


Tesha has 4 more cards than Jo, so they now look like this.


Holly's
4 2 $=6$

## APPENDIX II

## A Sample Rubric

Here is a rubric from Mark Cassar's school. In this rubric the teacher decides whether a student has met the area of learning described in the "criteria" and includes feedback for the student on ways to improve. In this case the rubric also reflects a conversation the teacher had with the student to clarify understanding.

Toothpicks Problem (Patterning)
Assessment For/As Learning

| Criteria | 1 | 2 | 3 | 4 | Feedback |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Create, identify, extend patterns |  | $\checkmark$ |  |  | "How could you find out the number of toothpicks for the 6th term?" |
| Make a table of values for a pattern | / |  |  |  | A table of values (t-chart) will help you determine the pattern rule |
| Communicate math thinking in writing and pictures (communication and representation) \& oral |  |  | $\checkmark$ |  | Conversation with student "tell me about" 22 $\begin{array}{r}22 \\ 6 \\ \hline\end{array}$ |
| 1 = expectation not met; 2 = approaching expectation; <br> 3 = meets expectation; 4 = exceeds expectation <br> * I had a conference with "I am adding together to find the the student; her changes total amount." are noted on $\# 2 b$. |  |  |  |  |  |


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